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A Law-and-Economics Perspective on Cost-Sharing Rules for a Condo Elevator

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Abstract: How to enhance the maintenance, repair and improvement of condo buildings? We address this issue by focusing on the case of an elevator installation whose benefits are not uniform across units. We examine the link between majority approval and cost sharing. Relying on a cooperative game theory approach, we prove the coalitional stability of any cost allocation which is such that the unit shares are a non-decreasing function of the floor level. Second, we show that the two surplus allocations induced, respectively, by the de facto cost-sharing rule used in France and the equal cost-sharing rule may fail to be coalitionally stable. By insisting that the cost sharing must depend on the relative individual advantages provided by an improvement, French law increases the risk of disputes between neighbors, compared to other sharing rules.

Keywords: condominium law, cost allocation, property law, shapley value

1 Introduction

How to enhance the maintenance, repair and improvement of apartment buildings? In the new European Union member states this problem has come to the forefront because – due to the mass privatization and sale, or granting of apartments to former tenants – there are inadequacies in the law ensuring the continuous maintenance and renovation of common areas, and of utilities (see, e.g. Lujanen, 2010). The apartments themselves are generally in good condition, but the common areas are often poorly maintained. Old European Union member states are also interested in legislative solutions for home ownership problems in condominium buildings (hereafter condos) because of their wish to facilitate the modernization of condos, and remove outdated features.

In most of the European jurisdictions, the purchaser of an apartment acquires individual ownership of his apartment, co-ownership (joint ownership) of the

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common areas of the building, and membership of the co-owners' association. Membership of the co-owners' association entitles each co-owner to participate in the process of collective decision-making. A deed regulates the joint ownership and relates the allocation of costs for utilities and improvements. This allocation differs from one country to another and is largely determined by the law. The allocation of costs generally depends on the shares in the common parts (and on the utility of the expenses for each co-owner). These shares, in turn, are commonly based on the relative values (as to the creation of the condominium) or relative sizes of the units proportional to the total value or size of all the units in the scheme. Certain jurisdictions prefer to allocate share values equally among all unit owners.[1]

In some countries, however, the cost sharing of an improvement must also take into account the differences in the benefits that accrue to units. For instance, the Italian Civil Code provides that "where a particular area benefits different owners to varying extents, the costs are apportioned accordingly (CC art. 1123 par. 3)" (Van der Merwe, 2015:206). In France, the condominium statute (statute no. 65-557 of 10 July 1965), namely the Law on Apartment Ownership of Buildings, does not provide any precise method for sharing the cost of an improvement. It only specifies that the sharing of this cost among the co-owners has to be made in such a way that each of them pays "in proportion to the advantages" he will receive from the planned work. Since these advantages are improvement and/or building specific, the French condominium statute is rather vague as to the details of the cost sharing. In other countries, even if the co-owners usually bear the cost of an improvement in proportion to their shares in the common parts, other alternative agreements can exist (this is the case in Germany and the Netherlands).[2]

To the best of our knowledge, little is known on the Law-and-Economics of the cost sharing of an improvement in a condo.[3] A key reference is Barzel and Sass (1990). In this paper, the authors, after developing a general theory of the allocation of resources by voting, focus their analysis on owners' associations for newly developed condominiums. They do that because, in contrast both with political organizations in which the one-man-one-vote rule is used almost exclusively,

1 See Appendix A for an overview (based on Van der Merwe, 2015) of condominium laws in a sample of European countries.

2 See appendix A for more details.

3 See, e.g. Tracht (1999) for an overview of condominiums from a Law-and-Economics view point. See also Lueck and Miceli (2007). Coloma (2001) shows that the condominium is an institution which economizes transaction and governance costs (especially when the number of agents involved is high). De Geest (1992) focuses on the majority requirements most condos choose for deciding on maintenance and improvement projects.

and with the corporate voting context in which the one-share-one-vote rule dominates, condominium associations have, amongst others, two key features: first, “the assignment of voting rights among condominium owner associations is relatively diverse” (*ibid*., p. 747), and second, “benefits in condominium associations include both pecuniary and non-pecuniary elements, thereby allowing an analysis of the form of benefits on institutional structure” (*ibid*.). In their view, the costs and voting rights in condominium associations are determined by the condominium developer who maximizes the value of individual units. To minimize the cost of voting (i.e. wealth capture, transaction costs), it is best to match the shares with the benefits. For instance, if the benefits are uniform, a uniform share across condominium units minimizes the voting costs. Otherwise, “the more varied are the units’ areas within a condominium, the less the likelihood that uniform assessments will lead to near unanimous votes, and the greater the likelihood that assessments that are proportional to unit size will enhance unanimity” (*ibid*, p. 757). Using data on a sample of American condominium homeowners associations, Barzel and Sass obtain results, which, taken as a whole, strongly support their theory.

This paper places a new perspective on the link made by Barzel and Sass (1990) between unanimous voting (or more generally majority approval) and cost sharing, when the benefits of an improvement are not uniform across units. A necessary condition for a unanimous approval of an improvement is that no coalition of co-owners is better off by breaking away from the grand coalition of all the co-owners. We therefore consider the coalitional stability of different cost-allocation schemes. To do this, we concentrate on the case of an elevator installation. This is a typical example of an improvement that yields different benefits across units. As pointed out by Barzell and Sass (*ibid*., pp. 756–757), there are a lot of similar examples in condominium associations: a swimming pool or a tennis court generate benefits proportional to the number of swimming or tennis-playing residents in a unit; a central heating system generates benefits proportional to the area of a unit; maintaining architectural standards or keeping common areas attractive generates benefits proportional to a unit’s value and so forth.

To analyze the coalitional stability of cost-sharing rules for installing an elevator, we rely on cooperative game theory, the branch of game theory that studies how to distribute costs that are collectively incurred by a group of players when the players can form coalitions and engage in binding agreements. Cooperative game theory applications to Law-and-Economics are scant (for an introduction, see, e.g. Benoît and Kornhauser, 2002).[4] Recently, however, cooperative game

4 Early application of cooperative game theory to Law-and-Economics includes the study of the link between the core and the law (Bergstrom, 1975), the application of core theory to competition (Telser, 1994), and the analysis of the bankruptcy problem from the Talmud (Aumann and Maschler, 1985).

theory has been used to study the sharing of damages payments among several injurers when more than one person is liable for the loss suffered by the victim (see Ferey and Dehez, 2016a and 2016b), or a compensation scheme for data sharing within the European REACH legislation (Béal and Deschamps, 2016).[5]

Concerning the installation of an elevator, our result is twofold. First, we prove the coalitional stability of any cost allocation which is such that the unit shares are a non-decreasing function of the floor level. Second, we show that the surplus allocations induced by both the de facto cost-sharing rule used in France and the equal cost-sharing rule may fail to be coalitionally stable. By insisting that the cost sharing must depend on the relative individual advantages provided by an improvement, French law increases the risk of disputes between neighbors, compared to other sharing rules, and therefore can impede the modernization of condos.

This paper unfolds as follows. In Section 2, we present the model of an N-storey condo which plans the installation of an elevator and we study the coalitional stability of cost-allocation schemes that are a non-decreasing function of the floor level. In Section 3, we consider the stability of the surplus sharing induced by two specific sharing rules (the French de facto sharing rule and the equal sharing rule). Section 4 concludes the paper. The proofs of our main results are relegated to the appendix.

2 Cost sharing and coalitional stability

This section studies some conditions under which a cost sharing related to the individual/coalitional cost savings derived from joint production of a condo improvement is coalitionally stable. To do this, we consider an N-storey condo which plans the installation of an elevator. We assume that there is one apartment unit on each floor and that each co-owner (one co-owner for each apartment unit) has one vote. Further, we associate a co-owner with the floor storey of his apartment.

The cost of an elevator comprises the following elements:

- a fixed cost, which includes everything to be done on the ground floor: c_f,
- a unit cost of building a storey: c_s.

For instance, the cost of building a three-storey (not including the ground floor) elevator, with access to all the storeys, is equal to: $c_f + 3c_s$.

5 REACH (Registration, Evaluation, Authorization, and Restriction of Chemicals) is a regulation of the European Union adopted to improve the protection of human health and the environment from the risks that can be posed by chemicals.

Let a coalition S of co-owners be given. Assume that this coalition wants to install an elevator benefiting co-owners of the coalition only. The cost $c(S)$ of this installation, also called the stand-alone cost, is:

$$c(S) = \max_{z \in S} c_z, \tag{1}$$

where c_z is the cost of an elevator which reaches storey z. More precisely, we set $c_0 = 0$ and $c_z = c_f + zc_s$.

A *cost-allocation scheme* is a way to share the installation cost $c(\{1, \ldots, N\})$ of an elevator that benefits *all* the co-owners. More formally, a cost-allocation scheme $(\lambda_i)_i$ is a list of non-negative real numbers such that $\sum_{i=1}^{N} \lambda_i = 1$ and whereby co-owner i bears a share $\lambda_i c(\{1, \ldots, N\})$ of the installation cost.

Example 1. The elevator rule

In France, as in other countries, the cost sharing of an improvement among co-owners has to be made in such a way that each of them pays "in proportion to the advantages" he will receive from the planned work. Remarkably, the French condominium statute does not define these advantages, nor the meaning of proportion. French case law, however, makes it clear that, as concerns the installation of an elevator in a condo, the advantage of each co-owner can be measured either by the difference between the market value of his condo unit before the improvement and its market value after the improvement, or by the permanent increase in rent generated by the improvement.[6] Given the French condominium statute and the related case law, there is a de facto rule for sharing the installation cost of an elevator in a condo. We call this de facto rule the "elevator rule". In the elevator rule we compute the cost-sharing scheme $(\lambda_i)_i$ as follows (see, e.g. Quignard, 2005). First, we associate to each storey i a specific coefficient. More precisely:
- the coefficient of the first storey is 1,
- the coefficient of the second storey is 1.5,
- the coefficient of the third storey is 2, and so on and so forth.

We obtain the cost-sharing scheme by dividing each storey's coefficient by the sum of all the coefficients. In the case of an N-storey building, the coefficient of storey i is given by the next expression:

$$\lambda_i = \frac{1 + (i-1)\frac{1}{2}}{\Sigma}, \; i = 1, \ldots, N, \tag{2}$$

6 See, e.g. Court of Appeal's judgment no.1998/17322 of 10 February 2000.

where Σ is the sum of the coefficients, i.e.[7]

$$\Sigma = N + \frac{N(N-1)}{4}. \tag{3}$$

Therefore, the relative share associated with a given unit in a storey rises with the number of storeys below it. The higher the storey, the higher the increase in property value, and, therefore, the higher the relative cost share.[8] Thus, the elevator rule satisfies the proportionality requirement specified by the French condominium statute.

Example 2. The equal sharing rule

This is the simplest cost-sharing rule that one can think of. This rule (see, e.g. the case of the Netherlands in Appendix A) also applies whenever the apartment sizes are the same and the shares in the common areas depend on the relative sizes.

It is interesting to compare the share borne by a storey i with the elevator and the equal sharing rules. By using the expression of λ_i (eq. (2)) we can show that the share borne by storey i with the elevator rule is higher than, or equal to, its share with the equal sharing rule if, and only if, $\frac{N+1}{2} \leq i$. Thus the cost shares of the top floors are no higher with the equal sharing rule than with the elevator rule. Conversely, the shares of the lower storeys with the equal sharing rule are higher than or equal to that with the elevator rule. This is in line with intuition because the elevator rule implies that the higher the storey, the higher the share of the total cost.

We now introduce a notion of coalitional stability for our cost-sharing problem.

Definition 1. *A cost-allocation scheme $(\lambda_i)_i$ is coalitionally stable if the cost borne by each coalition S is lower than its stand-alone cost:*

$$\left(\sum_{i \in S} \lambda_i\right) c(\{1, \ldots, N\}) \leq c(S), \ \forall S \subseteq \{1, \ldots, N\}. \tag{4}$$

7 We have $\Sigma = \sum_{i=1}^{N}(1 + \frac{1}{2}(i-1)) = N + \frac{1}{2}\sum_{i=1}^{N}(i-1) = N + \frac{N(N-1)}{4}$.

8 In France, according to Quignard (2005:124), units in buildings with an elevator are worth up to fifty percent more compared to walk-up units (*ceteris paribus*).

Therefore a cost-allocation scheme is coalitionally stable if no coalition of co-owners is better off by breaking away from the grand coalition of all the co-owners.[9] Our first result is as follows

Proposition 1. *Let a cost-allocation scheme be such that the share of a storey is a non-decreasing function of its floor level, namely $\lambda_i \leq \lambda_j$ whenever $i < j$. Then this cost-allocation scheme is coalitionally stable.*

From this Proposition we deduce immediately that the elevator rule is coalitionally stable. This is because, with this rule, the share of a condo unit rises with its floor level. The coalitional stability property enjoyed by the elevator rule can explain why it is a de facto rule. Besides being rather simple to implement, the elevator rule also avoids conflict among the co-owners, while complying with the law. The same remark applies for the equal-sharing rule.

An alternative coalitionally stable cost-allocation rule is the Shapley value. This value has reasonable axiomatic foundations and can be simply computed.[10] In effect, our cost-sharing problem is structurally identical to airport cost games. This kind of games was introduced by Littlechild and Owen (1973) to study aiport landing charges. Following these authors we can show that the Shapley value is such that storey i bears a cost sh_i equal to:

$$sh_i = \frac{c_f}{N} + c_s\left(\sum_{k=1}^{i} \frac{1}{N-(k-1)}\right) \tag{5}$$

The interpretation of this expression is as follows. First, the fixed cost is shared equally among the N storeys. Second, the cost of the first storey, c_s, is also equally shared among all the storeys (since all the inhabitants need to reach at least this storey). Third, the cost of the second storey, which is also c_s, is shared among the $N-1$ remaining storeys (since all these storeys need to reach that storey), and so forth. We notice from eq. (5) that the cost share borne by storey i rises with its floor

9 In cooperative game theory language we say that the cost allocation is in the core of the cost-sharing game.

10 On the axiomatic foundations of the Shapley value, see, e.g. section 18.1 in Maschler et al. (2013:748).

level. It then follows from Proposition 1 that the Shapley value is also coalitionally stable.[11,12]

3 Surplus shares induced by cost-sharing rules

Any cost-sharing scheme generates a surplus sharing (at least implicitly). Indeed, assume that building an elevator increases the storey i's apartment value by b_i (where b_i is a positive real number). As it is often observed, the increase in the apartment value is an increasing function of the storey level. Thus we may assume that $b_j < b_i$ whenever $j < i$. For a storey i, the surplus sharing resulting from any cost-allocation rule $(\lambda)_i$ can be defined as $b_i - \lambda_i c(\{1, \ldots, N\})$. This is the difference between the increase in storey i' s apartment value which results from the elevator installation and the cost borne by this storey. Does the coalitional stability of a cost-sharing rule imply the coalitional stability of the induced surplus-sharing rule? We address this issue by paying attention to the details of the law regarding the decision to make an improvement.[13] In doing so, we focus on the French case, bearing in mind that our approach can easily be adapted to other countries.

To study the surplus sharing, we next define the function v giving the *surplus* of a nonempty coalition S of co-owners

$$v(S) = \max_{T \subseteq S} \left\{ \sum_{i \in T} b_i - c(T) \right\}, \tag{6}$$

where $c(T) = max\{c_z, z \in T\}$ (see eq. (1)).

In this expression we recall that the term c_z is the cost of an elevator which reaches storey z. Now let us set $v(\varnothing) = 0$. Since $\varnothing \subset S$, we see from eq. (6) that

11 For an alternative proof relying on the fact that airport games are concave, see Theorem 18.32 of Maschler et al. (2013:767).

12 Despite its attractive properties, the Shapley value does not seem to be used to share the cost of an improvement in a condo. It is, however, applied for solving cost-sharing problems in many concrete cases (see, e.g. Moretti and Patrone, 2008).

13 The idea of introducing law on a surplus-sharing problem appeared originally in Aivazian and Callen (1981) (see also Benoît and Kornhauser, 2002). Aivazian and Callen provide an example where taking law into account when defining the gain of a coalition implies that no surplus-sharing is coalitionally stable. These authors deduce from this fact that the Coase theorem breaks down since no agreement is stable. There is a wide-ranging discussion on the relevance of this conclusion (see, e.g. Magnan de Borgnier, 1986; Benoît and Kornhauser, 2002, Gonzales and Marciano, 2017, Gonzales et al., 2018).

$v(S)$ is nonnegative. This means in particular that a nonempty coalition S will not build an elevator if the whole benefit of the decision is lower than its cost.[14]

The use of function $v(.)$ makes sense only if all the parameters b_i are known by all the co-owners. This is at first sight a strong assumption.[15] Yet, co-owners often have precise knowledge of building prices in the immediate vicinity of their home, and in particular of the links between these prices and the storeys. While co-owners may not know the *exact* figures, their guesses may be close to these figures.[16] Therefore, function $v(.)$ seems to be a reasonable theoretical construction.

Now consider a coalition S in which the number of members is *smaller* than $(2/3)N$. According to article 25 of French statute each improvement project of this coalition can be defeated in a majority vote. Moreover according to article 30 of the statute, such an improvement project can be authorized or rejected by the High Court. A good example of the problem under discussion is given by the French Court of Cassation's judgment of 16 December 2009 (judgement no.09-12654), which authorized co-owners to carry out a project (that was earlier defeated by a majority vote) involving works affecting the common areas of a condo.

As there is no certainty with regard to the High Court decision we assume that a judge authorizes coalition S to carry out its project with probability $p(S)$, $p(S) \in [0, 1]$. We thus assume that the probability $p(S)$ depends on the coalition. For instance, this probability could increase with the size of the coalition.[17] The dependence of $p(.)$ on S might be known from a lawyer's advice.

There is no doubt the Courts can control other aspects of an improvement project, and notably the cost sharing. In this regard, the improvement project receiving a majority vote could result in a court case brought by the minority members. It could well be the case that a court rejects a project on the ground that the cost sharing does not depend enough on the advantages received by the co-owners. We abstract, however, from this issue, because it seems that the courts validate the proportionality requirement rather loosely. Indeed, insofar as the cost sharing satisfies the proposition that the cost share associated with a given unit rises with the number of storeys below it, the sharing is considered legal.[18]

14 In addition, we notice that a coalition S will not necessarily build an elevator with $\max_{z \in S} z$ storeys.

15 As noted by a referee, the values of b_i might be less known than the cost of the improvement.

16 We can practically assess each b_i by subtracting the expected sale value of the apartment i before the improvement from the actual sale value of an apartment similar to i in a building with an elevator.

17 We thank two referees for suggesting this possibility.

18 As an illustration, we can instance the Paris Court of Appeal's judgment no.1998/17322 of 10 February 2000 (quoted above in note 6). This judgement deals with the case of a condo in which

Taking law into account according to the above remarks leads us to redefine the gain of a coalition. Thus, we distinguish two cases, depending on whether the size of the coalition is higher than or equal to $2N/3$. When the size of a coalition is higher than or equal to $2N/3$, the gain of a coalition is $v(S)$. When the size of the coalition is lower than $2N/3$, we *define* the gain of the coalition as $p(S)v(S)$.[19] In this definition, we make the (rather) pessimistic assumption that the general assembly of co-owners will not approve a project proposed by a minority of co-owners. This is keeping with the definition of the so-called α-characteristic function (see, e.g. Friedman, 1986:187). While the stand-alone project of a minority of co-owners will not receive a majority approval, we recall that this project can nevertheless be authorized with probability $p(S)$ by the high court.

Building on the above discussion and denoting by $v^l(S)$ the expected gain of a coalition (or the value of the characteristic function for the coalition S), we have:

$$v^l(S) = \begin{cases} v(S) \text{ if } \#S \geq \frac{2N}{3}, \\ p(S)v(S) \text{ otherwise,} \end{cases} \tag{7}$$

where $\#S$ is the cardinal of coalition S.

Now we define a *surplus allocation* as a list of non-negative real numbers $(s_i)_i$ such that $\sum_{i=1}^{N} s_i = v^l(\{1, \ldots, N\}) = v(N)$. Adapting the definition of coalitional stability used for a cost allocation to the case of a surplus allocation, we have:

Definition 2. *A surplus allocation $(s_i)_i$ is coalitionally stable whenever each coalition receives a payoff no lower than the surplus obtained when the stand-alone project is realized:*

$$\sum_{i \in S} s_i \geq v^l(S), \ \forall S \subseteq N. \tag{8}$$

We have the following result:

Proposition 2. *There exists a coalitionally stable surplus allocation.*

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an elevator has been installed. In this condo, there were two floors (ground floor and first floor) duplex apartments, each with internal staircase and two separate entries (one on each floor). The Court held that such a specific duplex apartment had to pay a cost share lower than that of a classic first-floor apartment. Indeed, the increase in the value of the duplex was considered to be lower than the increase in the value of the classic first-floor apartment.

19 See, Maschler et al. (2013:660), for a definition of the gain of a coalition in terms of expected payoff. Our game is a simple instance of a stochastic cooperative game. For another simple example of a stochastic cooperative game, see Dinar et al. (2006).

This result ensures that the sharing of the surplus resulting from a condo installation can be unanimously agreed upon. Yet, French law is concerned not with the surplus sharing resulting from an improvement, but with the cost sharing of this improvement. As in other laws, the requirement that this cost sharing be proportional to the advantages refers to an implicit concern with the fairness of the surplus sharing: the co-owners who benefit the most from the improvement must also bear the highest part of its cost. How does this concern affect the stability of the implicit surplus-sharing?

To tackle this issue first recall that for a storey i, the surplus sharing resulting from any cost-allocation rule $(\lambda)_i$ is $b_i - \lambda_i c(\{1,\ldots,N\})$. To be coalitionally stable the surplus sharing resulting from *any* sharing rule must satisfy the following condition

$$\sum_{i\in S} b_i - \left(\sum_{i\in S}\lambda_i\right) c(\{1,\ldots,N\}) \geq v^l(S),\ \forall S \subseteq \{0,\ldots,N\}. \tag{9}$$

As we shall see, this condition may not be satisfied for a given coalition S. Indeed, though $v^l(S)$ is always non-negative, it is possible that $\sum_{i\in S} b_i - \left(\sum_{i\in S}\lambda_i\right) c(\{1,\ldots,N\})$ is negative. This happens when the share of the total cost borne by the coalition S is greater than the sum of the advantages obtained by this coalition. We give an example of this case below. This example illustrates the fact that the two distinct surplus-sharing rules induced, respectively, by the elevator rule and the equal-sharing rule may not be coalitionally stable.

Example 3

Consider a three-storey condo where the $c_s = 1$ and $c_f = 5$. Assume also that $b_1 = 1$, $b_2 = 7$ and $b_3 = 13$. The costs borne by the different coalitions are as follows: $c(\{1\}) = 6$, $c(\{2\}) = c(\{1,2\}) = 7$, $c(\{3\}) = c(\{1,3\}) = c(\{2,3\}) = c(\{1,2,3\}) = 8$. In this example the values of the surplus function and of the characteristic function are exactly the same, with the exception that $v^l(\{3\}) = p(\{3\})v(\{3\}) = 5p(\{3\})$. For the other individual coalitions we have: $v^l(\{1\}) = v(\{1\}) = v^l(\{2\}) = v(\{2\}) = 0$. As for the other coalitions, the surplus is always legally feasible (the two-thirds majority requirement being always satisfied). We thus have: $v^l(\{1,2\}) = 1$, $v^l(\{1,3\}) = 6$, $v^l(\{2,3\}) = 12$ and $v^l(\{1,2,3\}) = 13$.

Let us now compute the cost sharing according to the elevator rule.[20] We find that $\lambda_1 = 2/9$, $\lambda_2 = 1/3$, and $\lambda_3 = 4/9$. Storey 1 bears a cost equal to 16/9,

20 We can check that $\Sigma = 4.5$.

storey 2 a cost equal to 24/9 and storey 3 a cost equal to 32/9. The surplus sharing induced by the elevator consists in giving $b^n(S) = \sum_{i\in S} b_i - \left(\sum_{i\in S}\lambda_i\right) c(\{1,\ldots,N\})$ to each coalition S. Examining the coalitional stability condition (9), we find that this condition is satisfied for all the coalitions, except for the coalition $\{1\}$:

$$b^n(\{1\}) = \frac{-7}{9} < v^l(\{1\}) = 0, \tag{10}$$

$$b^n(\{2\}) = \frac{39}{9} > v^l(\{2\}) = 0, \tag{11}$$

$$b^n(\{3\}) = \frac{85}{9} > 5 \geq v^l(\{3\}), \tag{12}$$

$$b^n(\{1,2\}) = \frac{32}{9} > v^l(\{1,2\}) = 1, \tag{13}$$

$$b^n(\{1,3\}) = \frac{78}{9} > v^l(\{1,3\}) = 6, \tag{14}$$

$$b^n(\{2,3\}) = \frac{124}{9} > v^l(\{2,3\}) = 12, \tag{15}$$

$$b^n(\{1,2,3\}) = \frac{117}{9} = v^l(\{1,2,3\}) = 13. \tag{16}$$

As $b^n(\{1\}) = -\frac{7}{9} < 0 = v^l(\{1\})$, the surplus sharing induced by the elevator rule is not coalitionally stable.

In the three-storey condo example considered above the equal sharing-rule requires that each storey bears one-third of the cost of an improvement. To this equal sharing-rule corresponds the following surplus sharing: $b^n(\{1\}) = 1 - 8/3 = -5/3$, $b^n(\{2\}) = 7 - 8/3 = 13/3$ and $b^n(\{3\}) = 13 - 8/3 = 31/3$. Since $b^n(\{1\}) < 0$, the surplus sharing induced by the equal sharing rule is not coalitionally stable.

Instead of sharing the surplus indirectly (from a given cost allocation), the co-owners could share it *directly*, notably by relying on the Shapley value. In the case of a three-storey condo, the surplus sharing corresponding to the Shapley value is given by the following expression:

$$sh_1(v^l) = \frac{pv(\{1\})}{3} + \frac{v(\{12\}) - pv(\{2\}) + v(\{13\}) - pv(\{3\})}{6} + \frac{v(\{123\}) - v(\{23\})}{3}, \tag{17}$$

$$sh_2(v^l) = \frac{pv(\{2\})}{3} + \frac{v(\{12\}) - pv(\{1\}) + v(\{23\}) - pv(\{3\})}{6} + \frac{v(\{123\}) - v(\{13\})}{3}, \tag{18}$$

$$sh_3(v^l) = \frac{pv(\{3\})}{3} + \frac{v(\{13\}) - pv(\{1\}) + v(\{23\}) - pv(\{2\})}{6} + \frac{v(\{123)\} - v(\{12\})}{3}. \tag{19}$$

However, the Shapley value of v^l is not always coalitionally stable. This result is illustrated in the example below.[21]

Example 4

Assume that: $c_s = 1$, $c_f = 5$, $b_1 = 1$, $b_2 = 8$, $b_3 = 11$ and $p = 0$. We thus have: $v(\{1\}) = 0$, $v(\{2\}) = 1$, $v(\{3\}) = 3$, $v(\{1,2\}) = 2$, $v(\{1,3\}) = 4$, $v(\{2,3\}) = 11$, $v(\{1,2,3\}) = 12$. Then we can check that the Shapley value of v^l is not in the core associated to v^l. Indeed, we can check that $sh_2(v^l) = 29/6$ and $sh_3(v^l) = 35/6$, and thus $sh_2(v^l) + sh_3(v^l) = 64/6 < v(\{23\}) = v^l(\{2,3\}) = 11$.

It is nevertheless possible to find a simple *direct* allocation rule s that is coalitionally stable.[22] This allocation is as follows:

$$s_1 = v(\{1\}), \tag{20}$$

$$s_2 = v(\{1,2\}) - v(\{1\}), \tag{21}$$

$$\ldots = \ldots \tag{22}$$

$$s_n = v(\{1,2,\ldots,N\}) - v(\{1,2,\ldots,N-1\}). \tag{23}$$

The above allocation rule gives to each storey the increment between the expected wealth created by the joint actions of the lower storeys, and the wealth created as a result of joining these storeys.

We notice that the Shapley value of v is always coalitionally stable for v^l because, as indicated in the proof of Proposition 2, v is convex and $v^l(S) \leq v(S)$ for all $S \subseteq N$.

Example 5

To illustrate rule s consider the following case of a three-storey condo such that: $v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$, $0 < v(\{1,2\}) < v(\{1,2,3\})$. According to the allocation

21 The fact that the Shapley value is not coalitionally stable is due to the fact that our game is not always convex. This lack of convexity directly stems from our assumption that $p(S)$ is below 1 when the coalition includes a single storey. That the Shapley value is not always coalitionally stable is also noticed in the paper by Dinar et al. (2006). But the model and the results of the present paper are different from Dinar et al. *ibid*'s.

22 This allocation is used in the proof of Proposition 2 to show constructively that there exists a coalitionally stable allocation. It is directly inspired by the proof of the Theorem 17.55 in Maschler et al. (2013:p . 719).

rule, $s_1 = 0$, $s_2 = b_1 + b_2 - 2c_s - c_f$, and $s_3 = b_3 - c_s$. Storey 1 owner receives nothing since he cannot afford the installation of an elevator for himself only. This is in contrast to the coalition including the owners of both storeys 1 and 2. That is because storey 2 owner's net benefit $(b_2 - c_s)$ can finance a share of the setup cost $(c_f + c_s)$ that storey 1 owner cannot afford to pay himself. Since storey 2 owner is instrumental in making possible the installation of an elevator for the coalition $\{1, 2\}$ he receives the whole surplus. Storey 3 owner only receives $b_3 - c_s$ because this is exactly the increase in the coalitional surplus resulting from his joining the coalition including the owners of the two first storeys.[23]

4 Conclusion

In this paper, by focusing on the case of a condo elevator installation, we have shed new light on the problem of modernization of condos. We have shown that any cost allocation which is such that the unit shares are a non-decreasing function of the floor level is coalitionally stable. Since the benefits of an elevator are a non-decreasing function of the highest level reached by the elevator, any cost sharing proportional to the benefits is coalitionally stable. This result also sheds light on a de facto sharing rule used in France for installing an elevator in a condo by proving that this rule is coalitionally stable.

To each cost-sharing rule corresponds a surplus sharing rule, namely the sharing of the advantages (including cost savings and increases in the values of the apartments) resulting from the installation of an elevator. We have also studied the coalitional stabililty of these induced surplus shares. We have shown for both the de facto sharing rule used in France and the equal-sharing rule that the surplus shares induced by these rules may fail to be coalitionally stable.

That the surplus sharing induced by a stable cost sharing may not be coalitionally stable shows that while cost sharing can be unanimously agreed upon, this may not be the case for the surplus sharing. We thus contend that when co-owners are legally bound to pay more attention to the surplus-sharing scheme than to the cost-sharing scheme, it is less likely that a cost-sharing rule will avoid conflict.

23 There is no denying that the allocation rule is somewhat arguable. For instance, storey 2 owner cannot build an elevator for himself only. But he can do this with storey 1 owner. In this regard storey 1 owner contributes to financing the fixed cost. But this contribution is not taken into account by the sharing rule.

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A A brief overview of cost sharing in European condominium laws

Share in the common areas

In Belgium, the number of shares into the common areas is determined on the basis of three exhaustive criteria: the net surface area, the destination and location of the private area. This value needs to be set out in a reasoned report by a notary, a land surveyor, an architect or an estate agent.

In Italy, contributions are apportioned by the share value of each unit unless otherwise agreed or provided for in the by-laws of the scheme (CC art. 1123). The Civil Code also provides that where common areas only benefit a particular group of owners, only those owners must contribute towards their maintenance (CC art. 1123 par. 2). Where a particular area benefits different owners to varying extents, the costs are apportioned accordingly (CC art. 1123 par. 3).

In the Netherlands, each of the apartment owners must "participate for an equal part in the debts and costs which are for account of all apartment owners jointly pursuant to law or the internal arrangements, unless the internal arrangements provide for another proportion of participation." (Article 5:113).

In Spain, according to Chapter two, Section 3 of the Horizontal Property Act, "An assessment quota shall be allocated to every unit in relation to the value of the building and expressed as a percentage of said value. Said quota shall serve as a coefficient to determine individual unit shares in the expenses and benefits of the community." [24]

In Germany, all co-owners must bear the costs incurred for maintaining/refurbishing the common property in proportion to their co-ownership quota, i.e. in proportion to the relative size of their apartments, if no alternative agreement exists (§16 par. 2 of the Condominium Ownership Act (Wohnungseigentumsgesetz, WEG)).

In France, the system of apartment ownership (*copropriété*) is governed by the statute 65-557 of 10 July 1965. As concerns the decisions on improvements

24 The basis for determining the quota is the usable area of each unit in relation to the building as a whole, its internal or external location, its situation and the reasonable foreseeable use of common elements and services.

in a condo, several articles of this law play a key role. Article 5 of the law states that “the share of the common areas pertaining to each lot is proportionate to the value of each private area in relation to the value of all said areas, as the value resulting from the establishment of the property, the substance and the situation of the lots without regards to their use”. Article 30 of the aforesaid law states that the general assembly of the co-owners establishes the distribution of the expenses for the improvement works “in proportion to the advantages that will result from the planned works for each of the co-owners, except when taking into account the consent of some of them to bear a higher proportion of the expenses”. According to case law the “advantage” is the increase in the apartment value stemming from the improvement. The 1965 statute (implemented by Decree no. 67-223 of 17 March 1967) has had several amendments, the last of which is the statute no. 2014-366 of 24 March 2014, which, like the laws that preceded it, makes various technical adjustments to certain provisions, such as reducing the majority required for works of preservation.

Majority rules

Decisions on improvements are usually taken by the members of the association of co-owners. The majority rules differ from one country to another.

In Belgium, the decisions are taken by a majority of the members of the association representing at least four fifths of the votes. Each co-owner has a number of votes which corresponds to his share in the common parts.

In Italy, A 50% quorum of all the owners and value of the building is required to carry out extraordinary and onerous repairs or maintenance works and other improvements.

In the Netherlands, the decisions on improvement work are taken by a majority of the members of the association representing at least two-thirds of the votes.

In Germany, the installation of a lift is considered as an improvement rather than mere maintenance and, therefore, a certain degree of approval (a qualified resolution), on the part of the owners must be attained in order to give effect to such measures.

In England and Wales, what needs to be done, in terms of repairs, service charges and so on, is decided by majority.

In France, Article 26 of the above law states that decisions concerning the works involving improvements “are taken by a majority of the members of the management association representing at least two thirds of the votes”. Article 30 of the law states that the general assembly of the co-owners establishes, with the same majority, the distribution of the expenses for the improvement works “in proportion to the advantages that will result from the planned works for each of the co-owners, except when taking into account the consent of some of them to bear a higher proportion of the expenses". In addition, article 30 of the aforesaid law states that, when the general assembly refuses the authorization defined in article 25, "any co-owner or group of co-owners can be authorized by the courts to execute any improvement works", such as the transformation of one or several existing facilities.

B Proof of the Propositions

Proof of Proposition 1

Proof. Assume that the Proposition is false. Therefore there exists a coalition S of co-owners such that:

$$\left(\sum_{i \in S} \lambda_i\right) c(\{1, \ldots, N\}) > c(S), \tag{24}$$

where: $c(S) = c_f + I(S)c_s$ and $c(\{1, \ldots, N\}) = c_f + Nc_s$, and $I(S) = \max_k\{k \in S\}$. The above inequation can be written as

$$\sum_{i \in S} \lambda_i > \frac{c_f + I(S)c_s}{c_f + Nc_s}, \tag{25}$$

or

$$c_f \left(\sum_{i \in S} \lambda_i - 1\right) + c_s \left(N \sum_{i \in S} \lambda_i - I(S)\right) > 0. \tag{26}$$

A necessary condition for the above inequation to be satisfied is

$$N \sum_{i \in S} \lambda_i - I(S) > 0. \tag{27}$$

Assume then that $\sum_{i \in S} \lambda_i > I(S)/N$. From this inequation, we conclude that $I(S) < N$. Now, since the shares λ_i are non-decreasing with i, we have:

$$\#S \times \lambda_{I(S)} > \frac{I(S)}{N}, \tag{28}$$

where $\#S$ is the cardinal of S. Since $\#S \leq I(S)$, it follows that:

$$\lambda_{I(S)} > \frac{1}{N}. \tag{29}$$

But then, using again the assumption that the shares λ_i are non-decreasing with respect to i, we have

$$\sum_{i=I(S)+1}^{N} \lambda_i + \sum_{i \in S} \lambda_i > \frac{N - I(S)}{N} + \frac{I(S)}{N} = 1. \tag{30}$$

This is a contradiction. □

Proof of Proposition 2

Proof. Let us first show that there is a coalitionally stable allocation to the surplus-sharing problem associated to v. This surplus-sharing problem corresponds to what is called an airport profit game in the cooperative game theory literature.[25] Now from Hou and Drissen (2013) we know that airport profit games are convex. It then follows from Theorem 17.55 of Maschler et al. (2013:719) that there always exists a coalitionally stable surplus allocation. In particular, Maschler et al. (*ibid*) shows that the allocation

$$\begin{aligned}
s_1 &= v(\{1\}), \\
s_2 &= v(\{1,2\}) - v(\{1\}), \\
\ldots &= \ldots \\
s_n &= v(\{1,2,\ldots,N\}) - v(\{1,2,\ldots,N-1\}).
\end{aligned}$$

is coalitionally stable (for the core associated to v). Let us show that this allocation is also coalitionally stable for the surplus-sharing problem associated to v^l. We know that the condition $\sum_{i \in S} s_i \geq v(S)$ hold for each $S \subseteq N$. Since $v^l(S) = p(S)v(S) \leq v(S)$ for all S we also have: $\sum_{i \in S} s_i \geq v^l(S)$. Moreover, $v^l(\{1,\ldots,N\}) = v(\{1,\ldots,N\})$. Therefore, the allocation (s_i) is coalitionally stable (for the core associated to v^l). The above argument applies for any allocation which is stable with respect to the core associated to v. □

25 The notion of airport profit game appears in Littlechild and Owen (1977:92), and later on in Brânzei et al. (2006).

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